ON THE TRACKING PROBLEM

(K ZADACHE PRESLEDOVANIIA)

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1. We assume that the behavior of the tracking object M is described by the equations

$$\frac{dx_j}{dt} = \sum_{k=1}^{r} a_{jk}(t)\, x_k + c_j(t)\, u(t) \qquad (j = 1, \ldots r) \tag{1.1}$$

and the behavior of the tracked object N by the equations

$$\frac{dy_j}{dt} = \sum_{k=1}^{r} b_{jk}(t)\, y_k + g_j(t)\, v(t) \qquad (j = 1, \ldots, r) \tag{1.2}$$

or in vector form

$$\frac{dx}{dt} = A(t)\, x + c(t)\, u(t), \qquad \frac{dy}{dt} = B(t)\, y + g(t)\, v(t) \tag{1.3}$$

The vectors $x(x_1, \ldots, x_r)$ and $y(y_1, \ldots, y_r)$ satisfy the initial conditions

$$x_1(0) = x_{10}, \ldots, x_r(0) = x_{r0}, \qquad y_1(0) = y_{10}, \ldots, y_r(0) = y_{r0}, \tag{1.4}$$

and the control functions $u(t)$ and $v(t)$ satisfy the conditions

$$|u(t)| \leqslant m, \quad |v(t)| \leqslant n \tag{1.5}$$

The coefficients $a_{jk}(t)$, $b_{jk}(t)$, $c_j(t)$, $g_j(t)$ are assumed to be functions which are differentiable r times. We shall also assume that the control function $v(t)$ is already known when the function $u(t)$ is chosen.

In [1,2] the following problem is formulated: If the equations

$$x_1(t_1) = y_1(t_1), \ldots, x_r(t_1) = y_r(t_1)$$

are satisfied at time $t = t_1$, then t_1 is called the instant of interception. Let the control functions $v(t)$ and $u(t)$ be given. The lowest positive value of t for which interception takes place is denoted by T_{uv}. We set

$$T_v = \min_u T_{uv} \qquad T^\circ = \max_v T_v = \max_v \min_u T_{uv}$$

It is required to find a pair of control functions $u^0(t)$, $v^0(t)$ (these control functions are said to be optimal) for which $T_{u^0 v^0} = T^0$. The value of T^0 can be used as a criterion in the choice of the parameters which define the motion of the object M. The necessary conditions which must be satisfied by $u^0(t)$, $v^0(t)$ are given in [1,2]. However, an efficient method of finding the optimal control functions has not yet been shown. For this reason it is of interest to have estimates of the length of time after which interception is ensured for any behavior of the object N. The present note describes a method of finding one such estimate for a definition of interception which is slightly different from the one used in [1]. Let some numbers $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ be given. By the interception time t_1 we shall mean a time $t_1 > 0$ for which the conditions

$$|x_1(t_1) - y_1(t_1)| \leqslant \varepsilon_1, \qquad |x_2(t_1) - y_2(t_1)| \leqslant \varepsilon_2 \tag{1.6}$$

are satisfied.

The set L of points T, which is included in $[0, \infty)$, is defined in the following manner: If $T \in L$, then for any control function $v(t)$ there is at least one function $u(t)$ such that on $v(t)$ and $u(t)$ there will be an interception at time T.

We shall describe below a method for determining whether a point T belongs to the set L.

For any instant T of L and any control function $v(t)$ a method is indicated for determining the control function $u(t)$ which will produce interception at time T.

It should be noted that the case in which the behavior of the tracked object is not known in advance is the more interesting one. However, there are problems for which the above formulation is valid.

The solutions of Equations (1.1) and (1.2), as is known, may be represented in the form

$$x_j(T) = x_j^0(T) + \int_0^T K_j(\tau) u(\tau)\, d\tau, \qquad y_j(T) = y_j^0(T) + \int_0^T G_j(\tau) v(\tau)\, d\tau \tag{1.7}$$

$$x_j^0(T) = \sum_{i=1}^r \varphi_{ij}(T) x_{i0}, \qquad K_j(\tau) = \sum_{i=1}^r \varphi_{ij}(T)(\psi_i(\tau), c(\tau)) \qquad (j = 1, \ldots, r)$$

Here the vectors $\varphi_i(\varphi_{i1}, \ldots, \varphi_{ir})$ form a fundamental system of solutions of the homogeneous equation

$$\frac{d\varphi}{dt} = A(t)\,\varphi \tag{1.8}$$

with the initial conditions $\varphi_{ij}(0) = \delta_{ij}$, and the vectors $\psi_i(\psi_{i1}, \ldots, \psi_{ir})$ form a fundamental system of solutions of equation

$$\frac{d\psi}{dt} = -A^*(t)\,\psi \tag{1.9}$$

with the initial conditions $\psi_{ij}(0) = \delta_{ij}$, where δ_{ij} is the Kronecker delta. Similar formulas apply to $y_j^{\circ}(T)$, $G_j(\tau)$.

The conditions for interception (1.6) at time T are of the form

$$\left| \Delta_j(T) + \int_0^T G_j(\tau)\,v(\tau)\,d\tau - \int_0^T K_j(\tau)\,u(\tau)\,d\tau \right| \leqslant \varepsilon_j$$

$$\Delta_j(T) = y_j^0(T) - x_j^0(T) \qquad (j = 1, 2) \tag{1.10}$$

The sets of control functions $u(t)$ and $v(t)$, respectively, satisfying the conditions

$$\int_0^T K_1(\tau)\,u(\tau)\,d\tau = a, \qquad \int_0^T G_1(\tau)\,v(\tau)\,d\tau = b$$

will be denoted by $U(a)$, $V(b)$. For any a in $[-A_T, A_T]$, b in $[-B_T, B_T]$, we can find control functions $u(t)$, $v(t)$ belonging to these sets;

$$A_T = m\int_0^T |K_1(\tau)|\,d\tau, \qquad B_T = n\int_0^T |G_1(\tau)|\,d\tau \tag{1.11}$$

We set

$$\alpha^+(a) = \sup_u \int_0^T K_2(\tau)\,u(\tau)\,d\tau, \quad \alpha^-(a) = \inf_u \int_0^T K_2(\tau)\,u(\tau)\,d\tau, \quad u \in U(a$$

$$\beta^+(b) = \sup_v \int_0^T G_2(\tau)\,v(\tau)\,d\tau, \quad \beta^-(b) = \inf_v \int_0^T G_2(\tau)\,v(\tau)\,d\tau, \quad v \in V(b)$$

If $T \in L$, then the conditions

$$\Delta_1 + B_T - A_T \leqslant \varepsilon_1, \quad \Delta_1 - B_T + A_T \geqslant -\varepsilon_1 \tag{1.12}$$

are satisfied.

For example, suppose that the first of these conditions is violated; then for the control function

$$v(\tau) = n \operatorname{sign} G_1(\tau) \qquad (\tau \in [0, T])$$

we cannot find a function $u(\tau)$ for which there will be interception at time T. Let us assume that for a given T the conditions (1.12) are satisfied. Let $b \in [-B_T, B_T]$. We set

$$\gamma^+(b, \varepsilon_1) = \Delta_2 + \beta^+(b) - \sup_a \alpha^+(a)$$
$$\gamma^-(b, \varepsilon_1) = \Delta_2 + \beta^-(b) - \inf_a \alpha^-(a)$$
$$a \in [\Delta_1 + b - \varepsilon_1, \Delta_1 + b + \varepsilon_1] \cap [-A_T, A_T]$$

In order that T should be an element of L, it is necessary and sufficient that for any b in $[-B_T, B_T]$ the inequalities

$$\gamma^+(b, \varepsilon_1) - \varepsilon_2 \leqslant 0, \ \gamma^-(b, \varepsilon_1) + \varepsilon_2 \geqslant 0 \tag{1.13}$$

be satisfied.

Let one of these conditions, say, the first, be violated, and let

$$\Delta_2 + \beta^+(b_1) - \sup_a \alpha^+(a) > \varepsilon_2, \qquad a \in [\Delta_1 + b_1 - \varepsilon_1, \Delta_1 + b_1 + \varepsilon_1]$$

hold for some b_1.

Then for the function $v(\tau)$ on which $\beta^+(b_1)$ is reached (we shall show later a method for constructing such a function), we cannot find a control function $u(\tau)$ which simultaneously satisfies the two conditions (1.12) and (1.13). The possibility of an effective verification of these conditions will be considered in Section 2.

Let $a \in [-A_T, A_T]$, $b \in [-B_T, B_T]$ be given. The symbols $u^+(\tau, a)$, $u^-(\tau, a)$, $v^+(\tau, b)$, $v^-(\tau, b)$ will denote functions of τ on which we have $\alpha^+(a)$, $\alpha^-(a)$, $\beta^+(b)$, $\beta^-(b)$. We set

$$K(\tau) = \frac{K_2(\tau)}{K_1(\tau)}, \qquad G(\tau) = \frac{G_2(\tau)}{G_1(\tau)}$$
$$K^+ = \sup_\tau K(\tau), \quad K^- = \inf_\tau K(\tau), \quad G^+ = \sup_\tau G(\tau), \quad G^- = \inf_\tau G(\tau) \quad (\tau \in [0, T])$$
$$I(x, y) = m \int\limits_{\sigma(x, y)} |K_1(\tau)| \, d\tau, \qquad E(x, y) = n \int\limits_{\delta(x, y)} |G_1(\tau)| \, d\tau$$

Here $\sigma(x, y)$, $\delta(x, y)$ are sets belonging to $[0, T]$ and such that if $\tau \in \sigma(x, y)$, then $y > K(\tau) > x$, and similarly, if $\tau \in \delta(x, y)$, then $y > G(\tau) > x$.

In what follows we shall need the following properties of the functions $K(\tau)$, $G(\tau)$. Each of the equations

$$K(\tau) = d, \ G(\tau) = d$$

where d is an arbitrary number, can have only a finite number of roots in $[0,\ T]$.

A sufficient condition for this is the independence of the vectors $c_1^0(\tau),\ \ldots,\ c_r^0(\tau)$ and $g_1^0(\tau),\ \ldots,\ g_r^0(\tau)$, respectively (see, for example, [2]), where

$$c_1^0(\tau) = c(\tau), \qquad c_j^0(\tau) = -A(\tau)\, c_{j-1}^0 + \frac{dc_{j-1}^0(\tau)}{d\tau} \qquad (j = 1, \ldots, r)$$

$$g_1^0(\tau) = g(\tau), \qquad g_j^0(\tau) = -B(\tau)\, g_{j-1}^0(\tau) + \frac{dg_{j-1}^0(\tau)}{d\tau} \qquad (j = 1, \ldots, r)$$

The proof of this fact is similar to the proof of Theorem 15 of Section 3 of [2].

We shall also show that $K(\tau)$ cannot have a discontinuity of the first kind in $(0,\ T)$. If at the point τ_1 the function $K(\tau)$ has a discontinuity of the first kind, then $K_1(\tau_1) = 0$, $K_2(\tau_1) = 0$.

It follows from (1.7) that

$$K_2^{(l)}(\tau) = \sum_{i=1}^{r} \varphi_{i2}(T)\,(\psi_i(\tau),\ c_{l+1}^0(\tau)) = \Big(\sum_{i=1}^{r} \varphi_{i2}(T)\,\psi_i(\tau),\ c_{l+1}^0(\tau)\Big) \ (l = 1, \ldots\ r-1)$$

We now note that at the point τ_1 at least one of the first $r - 1$ derivatives of the function $K_2(\tau)$ must be different from zero, for otherwise the vector

$$E = \sum_{i=1}^{r} \varphi_{i2}(T)\,\psi_i(\tau_1)$$

vanishes, since the system of equations

$$(E,\ c_l^0(\tau_1)) = 0 \qquad (l = 1, \ldots, r)$$

can have only a trivial solution because the vectors $c_1^0(\tau_1),\ \ldots,\ c_r^0(\tau_1)$ are linearly independent. But if the vector E is a zero vector, then since the vectors $\psi_i(\tau_1)$ form a fundamental system, all the numbers $\varphi_{12}(T),\ \ldots,\ \varphi_{r2}(T)$ must vanish; this is impossible, since the Wronskian of the system (1.8) would then vanish at the point T.

The function $K_1(\tau)$ has similar properties. Let

$$K_1(\tau_1) = K_1'(\tau_1) = \ldots = K_1^{(s-1)}(\tau_1) = K_2(\tau_1) = K_2'(\tau_1) = \ldots\ K_2^{(s-1)}(\tau_1) = 0$$

be true at the point τ_1 and let at least one of the values $K_1^{(s)}(\tau_1)$, $K_2^{(s)}(\tau_1)$ be different from zero.

By L'Hospital's rule

$$\lim_{\tau\to\tau_1}\frac{K_2(\tau)}{K_1(\tau)}=\lim_{\tau\to\tau_1}\frac{K_2^{(s)}(\tau)}{K_1^{(s)}(\tau)}$$

Since $s \leqslant r-1$, it follows that $K_2^{(s)}(\tau)$ is a continuous function, and if $K_1^{(s)}(\tau_1) \neq 0$, then

$$\lim_{\tau\to\tau_1+0}\frac{K_2^{(s)}(\tau)}{K_1^{(s)}(\tau)}=\lim_{\tau\to\tau_1-0}\frac{K_2^{(s)}(\tau)}{K_1^{(s)}(\tau)}$$

It was shown in [3] that

$$u^+(\tau,a)=m\,\mathrm{sign}\,K_1(\tau)\qquad \text{if}\quad \tau\in\sigma(y_0(a),K^+)$$
$$u^+(\tau,a)=-m\,\mathrm{sign}\,K_1(\tau)\qquad \text{if}\quad \tau\in\sigma(K^-,y_0(a))$$

where $y_0(a)$ is a root of the equation

$$I(K^-,y)=\frac{A_T-a}{2},\qquad y\in[K^-,K^+] \tag{1.14}$$

$$u^-(\tau,a)=-m\,\mathrm{sign}\,K_1(\tau)\qquad \text{if}\quad \tau\in\sigma(y_1(a),K^+)$$
$$u^-(\tau,a)=m\,\mathrm{sign}\,K_1(\tau)\qquad \text{if}\quad \tau\in\sigma(K^-,y_1(a))$$

where $y_1(a)$ is a root of the equation

$$I(y,K^+)=\frac{A_T-a}{2},\qquad y\in[K^-,K^+] \tag{1.15}$$

We replace the quantities u, m, K, σ, y, A, a, I in the above formulas by v, n, G, δ, x, B, b, E, respectively, to obtain the expressions for $v^+(\tau, b)$, $v^-(\tau, b)$. From the above mentioned properties of the function $K(\tau)$ it follows that the left-hand parts of (1.14) and (1.15) are strictly monotonic continuous functions of y. Since

$$a\in[-A_T,A_T],\qquad I(K^-,K^-)=0,\qquad I(K^-,K^+)=A_T$$

it follows that Equations (1.14) and (1.15) have unique solutions. Let us consider the time T for which the conditions (1.12) and (1.13) are satisfied. Let the control function $v(\tau)$ be given for $[0, T]$. We shall construct the control function $u(\tau)$ for which there will be interception at time T. We shall also prove thereby that the conditions (1.12) and (1.13) are necessary and sufficient conditions for T to be an element of L. We set

$$n_1(v)=\Delta_1+\int_0^T G_1(\tau)\,v(\tau)\,d\tau,\qquad n_2(v)=\Delta_2+\int_0^T G_2(\tau)\,v(\tau)\,d\tau$$

From the definition (1.11) of B_T it follows that

$$\left| \int_0^T G_1(\tau) v(\tau) d\tau \right| \leqslant B_1$$

By virtue of (1.12) the functions $\alpha^+(a)$, $\alpha^-(a)$, considered as functions of a, are defined on $[n_1(v) - \varepsilon_1,\ n_1(v) + \varepsilon_1] \cap [-A_T,\ A_T]$.

From the above properties of the function $K(\tau)$ it follows that $\alpha^+(a)$ and $\alpha^-(a)$ are continuous in $[-A_T,\ A_T]$. We shall denote by a^+ and a^- the points at which we have, respectively,

$$\sup_a \alpha^+(a),\ \inf_a \alpha^-(a) \quad \text{if} \quad a \in [n_1(v) - \varepsilon_1,\ n_1(v) + \varepsilon_1] \cap [-A_T,\ A_T]$$

We shall show that

$$u(\tau) \equiv u^+(\tau, a^+) \quad \text{if} \quad n_2(v) \geqslant \alpha^+(a^+)$$

In fact, $u^+(\tau,\ a^+) \in U(a^+)$ and, consequently, the first of the conditions (1.6) is satisfied. The second condition of (1.6) is satisfied, since, by virtue of (1.13)

$$\Delta_2 + \beta^+(n_1(v)) - \alpha^+(a^+) \leqslant \varepsilon_2$$

and at the same time

$$\Delta_2 + \beta^+(n_1(v)) \geqslant n_2(v) \geqslant \alpha^+(a^+)$$

Similarly it can be shown that

$$u(\tau) \equiv u^-(\tau, a^-) \quad \text{when } n_2(v) \leqslant \alpha^-(a^-)$$

Now let

$$\alpha^-(a^-) < n_2(v) < \alpha^+(a^+) \tag{1.16}$$

There are two possibilities:

1) At least one of the equations in a

$$\alpha^+(a) = n_2(v), \qquad \alpha^-(a) = n_2(v) \tag{1.17}$$

has a root in the interval $[n_1(v) - \varepsilon_1,\ n_1(v) + \varepsilon_1]$.

Let the first of these equations have a root at the point a_0; then

$$u(\tau) \equiv u^+(\tau, a_0)$$

If the second equation has a root at the point a_1, then

$$u(\tau) \equiv u^-(\tau, a_1)$$

2) Neither of Equations (1.17) has a root in $[n_1(v) - \varepsilon_1,\ n_1(v) + \varepsilon_1]$. In that case, by virtue of the condition (1.16) and the equalities

$$\alpha^-(A_T)=\alpha^+(A_T), \qquad \alpha^-(-A_T)=\alpha^+(-A_T)$$

we have the relation

$$\alpha^-(n_1(v))<n_2(v)<\alpha^+(n_1(v)) \tag{1.18}$$

We introduce the function

$$u_\lambda(\tau)=\lambda u^+(\tau, n_1(v))+(1-\lambda)u^-(\tau, n_1(v)) \qquad (0\leqslant\lambda\leqslant 1) \tag{1.19}$$

which depends on the parameter λ.

This function belongs to the set $U(n_1(v))$ for any value of λ in (1.19). In fact

$$|u_\lambda(\tau)|\leqslant\lambda|u^+(\tau, n_1(v))|+(1-\lambda)|u^-(\tau, n_1(v))|=m$$

$$\int_0^T K_1(\tau)u_\lambda(\tau)\,d\tau=\lambda\int_0^T K_1(\tau)u^+(\tau, n_1(v))\,d\tau+(1-\lambda)\int_0^T K_1(\tau)u^-(\tau, n_1(v))\,d\tau=n_1(v)$$

The equation

$$\lambda\alpha^+(n_1(v))+(1-\lambda)\alpha^-(n_1(v))=n_2(v) \tag{1.20}$$

which is linear in λ, has, by virtue of (1.18), a root

$$\lambda_0=\frac{n_2(v)-\alpha^-(n_1(v))}{\alpha^+(n_1(v))-\alpha^-(n_1(v))}$$

which belongs to (0, 1).

Thus

$$u(\tau)\equiv u_{\lambda_0}(\tau)$$

2. We now consider the computational side of the problem. The determination of $K_j(\tau)$, $G_j(\tau)$, $x_j^0(T)$, $y_j^0(T)$ reduces, as can be deduced from (1.7), to the calculation of the normal fundamental systems of solutions of Equations (1.8) and (1.9). The methods for finding these solutions by means of high-speed digital computers or analog computers are well known. The condition (1.12) is easily checked since A_T and B_T are calculated by the simple Formulas (1.11). Let us discuss the verification of the conditions (1.13). The function $\alpha^+(a)$ has not more than one extremum in the interval $[-A_T, A_T]$.

Indeed, from the definition of $y_0(a)$ and the restrictions imposed on the function $K(\tau)$ it follows that $y_0(a)$ decreases monotonically in the strict sense from the value K^+ to the value K^- when a varies from $-A_T$ to A_T. We shall show that

$$\frac{d\alpha^+(a)}{da}=y_0(a) \tag{2.1}$$

From the definition of $u^+(a)$ it follows that

$$\frac{\alpha^+(a+\Delta a)-\alpha^+(a)}{\Delta a}=\frac{2m}{\Delta a}\int\limits_{\omega(\Delta a)}K(\tau)K_1(\tau)\operatorname{sign}K_1(\tau)\,d\tau$$

where

$$\omega(\Delta a)=\sigma(y_0(a+\Delta a),\ y_0(a))$$

Applying the generalized mean value theorem and noting (1.14) that

$$m\int\limits_{\omega(\Delta a)}K_1(\tau)\operatorname{sign}K_1(\tau)\,d\tau=\frac{\Delta a}{2},\qquad y_0(a+\Delta a)<K(\tau)<y_0(a)\quad \text{if}\quad \tau\in\omega(\Delta a)$$

we obtain (2.1) by passing to the limit. Similarly

$$\frac{d\beta^+(b)}{db}=x_0(b),\qquad \frac{d\alpha^-(a)}{da}=y_1(a),\qquad \frac{d\beta^-(b)}{db}=x_1(b) \tag{2.2}$$

Here $x_0(b)$ is decreasing monotonically in the strict sense and $y_1(a)$ and $x_1(b)$ are increasing monotonically in the strict sense. In order to verify (1.13) we must calculate

$$\zeta^+(b)=\sup_a\alpha^+(a),\qquad a\in[\Delta_1+b-\varepsilon_1,\ \Delta_1+b+\varepsilon_1],\qquad b\in[-B_T,\ B_T]$$

If the function $y_0(a)$ is positive and does not vanish in the interval $(\Delta_1-B_T,\ \Delta_1+B_T)\cap[-A_T,\ A_T)$, then for a given b

$$\zeta^+(b)=\alpha^+(\Delta_1+b+\varepsilon_1)\quad \text{if}\quad \Delta_1+b+\varepsilon_1\leqslant A_T$$
$$\zeta^+(b)=\alpha^+(A_T)\quad \text{if}\quad \Delta_1+b+\varepsilon_1>A_T$$

If the function $y_0(a)$ is negative, then

$$\zeta^+(b)=\alpha^+(\Delta_1+b-\varepsilon_1)\ \text{when}\ \Delta_1+b-\varepsilon_1\geqslant-A_T,\ \zeta^+(b)=\alpha^+(-A_T)\ \text{when}\ \Delta_1+b-\varepsilon_1<-A_T$$

If $y_0(a)$ vanishes at the point $a^*\in[\Delta_1-B_T,\ \Delta_1+B_T]\cap[-A_T,\ A_T]$, then

$$\zeta^+(b)=\alpha^+(\Delta_1+b+\varepsilon_1)\qquad \text{if}\quad b\in[-B_T,\ a^*-\Delta_1-\varepsilon_1]$$
$$\zeta^+(b)=\alpha^+(a^*)\qquad \text{if}\quad b\in[a^*-\Delta_1-\varepsilon_1,\ a^*-\Delta_1+\varepsilon_1]$$
$$\zeta^+(b)=\alpha^+(\Delta_1+b-\varepsilon_1)\qquad \text{if}\quad b\in[a^*-\Delta_1+\varepsilon_1,\ B_T]$$

Similar relations are also readily written for

$$\zeta^-(b)=\inf_a\alpha^-(a),\qquad a\in[\Delta_1+b-\varepsilon_1,\ \Delta_1+b+\varepsilon_1],\qquad b\in[-B_T,\ B_T]$$

Thus the functions

$$\beta^+(b),\ \zeta^+(b),\ \beta^-(b),\ \zeta^-(b)$$

are very simple in form. The graphs of these functions, therefore, can be constructed with any desired degree of accuracy if we calculate the values of the functions and their derivatives at a comparatively small number of points.

Let us consider a method of calculating these functions at specified points. (We shall thereby also prove the possibility of an efficient check of the conditions (1.13).) A calculation of $\alpha^+(a)$ reduces to finding the function $u^+(a)$. A root $y_0(a)$ of Equation (1.14) is most conveniently found by some method of successive approximations, for example, the method of false position. Since the left-hand part of (1.14) is a monotonically increasing function of y, it follows that to determine $y_0(a)$ to a given degree of accuracy we need to calculate the value of $I(K^-, y)$ at a small number of points. To calculate $I(K^-, y)$ for a given y we must find the set $\sigma(K^-, y)$. The boundary points of this set are roots of the equation in τ

$$K(\tau) = y$$

As was pointed out above, this equation can have only a finite number of roots in $[0, T]$, and consequently $\sigma(K^-, y)$ consists of a finite number of intervals and isolated points.

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